Astronomy
TEST BOOKLET

Science Olympiad at the
University of Pennsylvania
February 22nd, 2020

Instructions to participants:

- *Do not open this exam or write on the answer sheet until instructed to do so.* Opening the exam early and any violation of standard testing protocol may incur a scoring penalty or disqualification.
- Please refrain from writing on this exam, and write all answers in the answer sheet. *Any marks elsewhere will not be scored.*
- Please write your team name and number on every page *of the answer sheet*. If any pages are lost, you will not receive credit for anything you wrote on them.
- You may take apart the exam, but make sure you turn in all materials at the end of the event.
- Question parts with * are worth 1 pt, ** are 2 pts, *** are 3 pts, etc.
- Each question has its total point values in parentheses
- Partial credit is given for answering part of a question
- Answers to math questions are accepted to within 10% of the correct answer
- **Tie breakers**:
  Section 1 - 3.c, 6.d, 1.b, 7.c
  Section 2 - 3.b, 4.h, 2.i

Section 1: DSOs/theory (65 pts.)

1. The object in image 4 was first detected in the CANDELS Ultra Deep Survey. (7)
    a. *What is the name of this object?
    b. *What does the acronym CANDELS stand for?
    c. **What is notable about this object's distance?
    d. ***This object is situated in the sky near a galaxy. How might the galaxy's positioning in front of the object affect its distance measurement? (In this case it was found that this effect was not significant)
2. Image 6 shows the galaxy that contains the first imaged black hole (BH). (5)
    a. *What is the Messier catalogue number of this galaxy?
    b. **What telescope technique did scientists use to get sufficiently fine resolution to help see the supermassive BH?
    c. **This technique relies on a different common analysis technique to gather information from the resulting fringes. It is also often used when analyzing waves and breaking them down to their base frequencies. What is this called?
3. Image 2 shows one of the farthest galactic clusters measured. (9)
    a. *I acquired this image by going into one of the sample image archives provided by JS9. This came in a file which uses one of the most common file types for transferring image data in astronomy. What type of image file does JS9 accept when accessing by proxy?
    b. *In 2009, this object was the farthest observed of its kind. What is its z value?
    c. ***There are at least seven ways to measure the mass of this cluster. Name three of them.
    d. **One of those ways involves measuring the effect on the Cosmic Microwave Background (CMB). Briefly explain how the cluster can affect the CMB.
    e. **What is the estimated mass of the cluster in solar masses?
4. The Bullet Cluster is sometimes called one of the most "interesting galaxy clusters in the universe" (7)

a. *Which image shows this cluster?

b. *This object isn't a single boring cluster. What is it really?

c. *Gravitational lensing around this cluster hints at the existence of what mysterious substance?

d. **There has been much discussion about the exact type of mysterious substance that this cluster demonstrates. If the widely accepted model for the rest of the universe was applied to this instance, the probability of an event like this occurring would actually be less than 1 in a million. This is because it is estimated that the two clusters were moving toward each other at very high velocities. Approximately what is that velocity?

e. **A model that indeed agrees with observations and is much more likely to result in this object is a type of modified gravity. What is the name of this model, first proposed in 1983?

5. This object was the first quasar to ever be identified. (6)

a. *Which image shows this object?

b. *How quickly is this object receding from Earth?

c. *What is the absolute magnitude of this object?

d. *What well-known stellar object has a similar apparent magnitude to this object's absolute magnitude?

e. **Extensive measurements of this object's broad emission-line region have been carried out since its discovery. What technique is primarily used to accomplish these measurements?

6. Image 5 shows a simulation of the evolution of a massive gas cloud in the early universe caused by the merger of two Milky-Way-mass galaxies. In particular, the chosen frame in this image is about 30 thousand years after the merger. (10)

a. *The authors of the paper from which this image was taken made this simulation to see if they can create direct-collapse black holes (DCBH) from a gas cloud. Which DSO is believed to be caused by this type of process?

b. *Aside from DCBHs, what other objects can possibly result from this situation?

c. **This type of simulation uses what is called "smooth particle hydrodynamics." The resolution used is 0.1pc. If the authors wanted to study “particle hydrodynamics,” why did they use such a relatively low resolution?

d. **This type of simulation has been done before, so to contribute to science, the authors needed to add variables to make a more realistic scenario than in previous simulations. In this case, they included the effects due to radiative cooling and the dynamics of internal opacity. Would DCBHs be more likely or less likely to form due to higher internal opacity? Why?

e. **Estimate the mass of one of the gas clouds resulting from the merger in the image.

f. **Does this estimated mass seem reasonable for a supermassive black hole? Is it more or less massive than the SMBH in the Milky Way?

7. This is a quasar that has been studied to help determine the distances to other quasars using its emitted light. (11)

   a. *Which image represents this DSO?

   b. *As mentioned earlier, this DSO was used as part of a survey that helped standardize the distance measurements to other galaxies. A similar technique is used with type Ia supernovae. What is the term for this type of object once used to measure distances?

   c. **There is a relationship between the luminosity of the quasar and the line width of certain emission lines for quasars that is represented by $W_\lambda \propto L^{-2/3}$. This is what astronomers sometimes use to determine the distances to far quasars. What is this relationship called?

   d. **The carbon IV (C IV) emission line is often used to calculate the luminosity using this relationship. What usually causes the (C IV) line to broaden after emission?

   e. **How many times ionized is carbon in (C IV)? What is its signature wavelength in nanometers?

   f. ***Another line often used from quasars is the Lyman-alpha line emitted by an electron going from n = 2 to n = 1 in hydrogen. As this wavelength travels through the universe, it loses energy and gets longer due to expanding space. When it comes in contact with a cloud of gas in the intergalactic medium, some of it gets scattered or absorbed by the gas, leaving a signature on the light at the wavelength it got scattered. Mapping the z values corresponding to the redshifts measured in the light that reached Earth lets astronomers calculate where in space the gas clouds are located. This technique then allows them to model a feature that would otherwise not have been noticed. What is this feature called?

8. Image 3 is a quasar (What a shocker). (10)

    a. *Which DSO does this image show?

    b. **What large-scale object does the blue line represent?

    c. **This object has its jet pointing directly towards the Earth. What is this type of quasar called? Within this type, what is its specific subtype?

    d. *The jet from the quasar is relativistic. What does this mean?

    e. **By what process is most of the emitted energy in the jet released? This process is closely tied with accelerating charged particles in magnetic fields.

    f. **Light observed coming from the jet is much more luminous than the light emitted in the rest frame of the jet. This is called relativistic beaming. Briefly explain why the light appears more luminous.

Section 2: Math (60 pts.)

1. A gaseous planet with 4 M_J (Jupiter masses) is orbiting a 2 $M_{\odot}$ neutron star with a period of 0.04 yr. (13)
    a. *What is the semimajor axis of the orbit in AU?
    b. **If the planet orbits with an eccentricity of e = 0.7, what is the farthest separation in the orbit in AU? What is the closest?
    c. *Assuming an average density of $1 g/cm^3$ and given 1 $M_J = 1.898*10^{27}$kg, what is the volume of the gas planet, in m^3? Assume it is a perfect sphere.
    d. *What is its radius, in AU?
    e. **Imagine that the planet is at periapsis. What is the ratio of the force on a particle on the point on the planet closest to the neutron star, F_1, to the force on a particle on the point farthest from neutron star, F_2 (i.e. F_1/F_2)?
    f. *The last question demonstrates a significant difference of force exerted by the neutron star on one side of the planet over the other. What is this type of interaction called?
    g. **Over a long period of time, these interactions dissipate kinetic energy by interacting with the rotating gas on the planet. However, because this is an isolated system, angular momentum must be conserved. So, while kinetic energy is lost, the orbital momentum remains the same. What type of orbit for a certain magnitude of momentum has the least amount of associated kinetic energy?
    h. **These types of forces vary with an inverse sixth power of distance relationship. How many times more significant is this effect at periapsis than apoapsis for this planet?
    i. *Due to the loss of energy, the planet will experience circularization, which means its orbital eccentricity will tend to e = 0. Trust me, I tried it in Universe Sandbox 2 and it works. Isn't this really, really cool? (yes/no)
2. A galaxy in a galaxy cluster is observed to have an Hα line measured at 700nm. Assume that H_0 = 70 km/s/Mpc. (11)
    a. *What is this galaxy's z (redshift) value?
    b. **How quickly is the galaxy receding from Earth, in km/s? Remember to use the relativistic versions of formulae if necessary

c. *Using Hubble's Law, how far away is this galaxy, in Mpc?

d. **Since this galaxy is in a galaxy cluster, applying Hubble's law directly to the galaxy yields an invalid value for distance. Assume velocity dispersion of galaxies in the cluster is 1500 km/s and that this galaxy is two standard deviations away from the mean so the galaxy has a larger velocity than the cluster. What is the velocity of the cluster's center of mass in km/s?

e. *How far away is this cluster in Mpc?

f. **Assuming virial equilibrium, the mass of the cluster can be calculated using the formula, $\frac{GM}{2R} \sim \sigma^2$ where σ is the velocity dispersion, R is radius, G is the gravitational constant, and M is mass. What is the mass of the cluster in solar masses, assuming a radius of 3.5Mpc? Make sure to pay attention to units.

g. *The mass-to-light ratio, $Y_\odot$, is defined as $M_\odot/L_\odot$. For the sun, $Y_\odot = 1$. For a typical galaxy cluster, $Y_\odot = 100$. Using this value, what is the power output of this galaxy cluster in solar luminosities?

h. *A higher $Y_\odot$ means less matter exists in the form of stars. What form does most of the mass take instead?

3. The orbital velocity of stars at different points in a galaxy is measured indirectly by observing their redshift. In particular, the stars 15 kpc from the core are found to have an orbital speed 1.5 times greater than those 5 kpc from the core. (8)

   a. **Let m_a be the mass of the galaxy within 5 kpc of the core; let m_b be the mass of the galaxy within 15 kpc of the core. Find the ratio m_b/m_a.

   b. **Let ρ_a be the average density (i.e. mass/area in the galactic plane) within 5 kpc of the core; let ρ_b be the average density of the galaxy **outside of 5 kpc and within 15 kpc** of the core. Find the ratio ρ_b/ρ_a.

   c. **Visual observations of stellar distribution have found the ratio ρ_b/ρ_a to be around 1/3. What does this imply about the distribution of dark matter in the galaxy? Explain.

   d. **Using the same definition of $Y_\odot$ as in question (2) part (g) and assuming $Y_\odot = 1$ for each individual star, would we expect the value to be higher or lower for a galaxy than a galaxy cluster? Why?

4. A Sérsic profile of a galaxy describes the intensity at varying radii from the center based on its geometry. It is given by:

$$I(R) = I_e e^{\left(\frac{1}{3}-2n\right)*\left(\left[\frac{R}{Re}\right]^{\frac{1}{n}}-1\right)}$$

where R_e is the half-light radius and I_e is the intensity there. The term “n” is the Sérsic index, which scales the rate that the intensity falls with radius. (10)

a. *Suppose we have an ordinary spiral galaxy with n = 1. At R = 2R_e, what is I(R) in terms of I_e? Evaluate the coefficient of I_e to obtain a decimal value.

b. *At R = 0.5R_e, what is I(R) in terms of I_e?

c. *Now suppose we have an elliptical galaxy with n = 4. At R = 2R_e, what is I(R) in terms of I_e?

d. *At R = 0.5R_r, what is I(R) in terms of I_e?

e. **From these calculations, would you expect stellar mass to be more evenly distributed throughout a spiral or an elliptical galaxy?

f. *Sometimes the intensity curves are described as “cuspy,” which means that if the intensity curve was mirrored across the center of the galaxy, it would produce a “Λ” shape at the top rather than a “∩” shape. Will a higher or lower Sérsic index curve be more cuspy?

g. *The masses of the supermassive black holes (SMBHs) at the centers of galaxies have been shown to be correlated to the Sérsic index through an empirical relation:

$$\log M_{BH} = 2.68 \log \frac{n}{3} + 7.82$$

where M_{BH} is in $M_{\odot}$. Given that the mass of the SMBH in the Milky Way is 2.6 million $M_{\odot}$, what would we expect the Sérsic index of our galaxy to be?

h. **Because there is a close empirical relationship between the mass of the black hole and the properties of the structure of its host galaxy, what can we infer about galactic evolution?

5. Gravitational waves are fun. (18)

a. *Are gravitational waves generally considered transverse or longitudinal?

b. **While EM waves are dipole waves, gravitational waves are what?

c. **The answer to the previous two questions means that space is stretching and compressing in how many perpendicular directions? How many of those directions are perpendicular to the direction of motion?

d. *Gravitational waves can be produced by an accelerating object relative to another, such as in a binary system. The power P of energy emitted in gravitational waves by a binary system is given by the equation:

$$P = \frac{32G^4}{5c^5r^5}\left(m_1m_2\right)^2\left(m_1 + m_2\right)$$

Where r is the orbital separation, c is the speed of light, G is the gravitational constant, and m_1 and m_2 are the masses of the two objects. What is the power, in Watts, of gravitational waves emitted by the Earth-Sun system for r = 1AU?

e. **Using some calculus (which you luckily don't have to do), we can get the equation for the time for a binary to release enough energy to merge:

$$t = \frac{5c^5}{256G^3}\frac{r^4}{(m_1m_2)(m_1+m_2)}$$

How long, in billions of years, would it take two 15 $M_\odot$ black holes separated by 0.1 AU to collide? 1 parsec?

f. **How long, in billions of years, would it take two 15 million $M_\odot$ black holes separated by 0.1 AU to collide? 1 parsec?

g. ***You should have gotten an unreasonably large number for 1 parsec for both (e) and (f). This should suggest that it should be impossible for even SMBHs to merge at such separations because it would take too long to release enough energy in the form of gravitational waves. This is known as the "final parsec problem." However, it is well-modeled and understood how the black holes reach the parsec separation from large distances. Briefly explain how it reaches this separation and why this process does not apply below 1pc.

h. *****Even though the final parsec should take too long to traverse, we still observe such mergers occurring in the universe. One possible explanation is the presence of a third SMBH that takes the energy from the other two and is flung out of the system at incredibly high speeds. What fraction of the speed of light would a third SMBH with 15 million $M_\odot$ be going if it took the potential energy from the two SMBHs to reduce their separation from 1pc to .01pc and converted it to kinetic energy? Ignore the energy the SMBH loses as it gains potential energy relative to the system and assume it starts at zero velocity in the frame of the galaxy.